

Matematica 2, Steger — Esercizi di integrazione multipla

1. Calcolare

$$\iiint_{0 \leq x \leq y \leq z \leq 1} 1 \, dx \, dy \, dz$$

con le coordinate cilindriche.

2. Calcolare

$$\begin{aligned} & \iiint_{0 \leq x \leq y \leq z \leq 1} 1 \, dx \, dy \, dz & \iiint_{0 \leq y \leq x \leq z \leq 1} 1 \, dx \, dy \, dz & \iiint_{0 \leq x \leq z \leq y \leq 1} 1 \, dx \, dy \, dz \\ & \iiint_{0 \leq z \leq x \leq y \leq 1} 1 \, dx \, dy \, dz & \iiint_{0 \leq y \leq z \leq x \leq 1} 1 \, dx \, dy \, dz & \iiint_{0 \leq z \leq y \leq x \leq 1} 1 \, dx \, dy \, dz \end{aligned}$$

con le coordinate sferiche.

3. Per costanti a , b , e c , calcolare

$$\iiint_{0 \leq x/a \leq y/b \leq z/c \leq 1} 1 \, dx \, dy \, dz$$

con le coordinate rettangolari e ancora con le coordinate sferiche.

4. Calcolare:

$$\begin{aligned} & \iiint_{(x-R)^2 + y^2 + z^2 \leq R^2} 1 \, dx \, dy \, dz \\ & \iiint_{x^2 + y^2 + (z-R)^2 \leq R^2} 1 \, dx \, dy \, dz \\ & \iiint_{(x-R)^2 + y^2 + z^2 \leq R^2} (x^2 + y^2 + z^2) \, dx \, dy \, dz \\ & \iiint_{x^2 + y^2 + (z-R)^2 \leq R^2} (x^2 + y^2 + z^2) \, dx \, dy \, dz \\ & \iiint_{(x-R)^2 + y^2 + z^2 \leq R^2} (x^2 + y^2 + z^2)^2 \, dx \, dy \, dz \\ & \iiint_{x^2 + y^2 + (z-R)^2 \leq R^2} (x^2 + y^2 + z^2)^2 \, dx \, dy \, dz \end{aligned}$$

5. Calcolare:

$$\iiint_{-\infty \leq x, y, z \leq \infty} e^{-x^2 - y^2 - z^2} \, dx \, dy \, dz$$

con le coordinate rettangolari, con le coordinate cilindriche, e con le coordinate sferiche. Si ricorda che $\int_{-\infty}^{\infty} e^{-t^2} \, dt = \sqrt{\pi}$.