ECONOMIC STATISTICS

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Module 2	Price Indices and Measuring Inflation
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Topic	The Index Numbers

INDEX NUMBERS

Simple Index Number

- It is the ratio of two values of the same phenomenon measured under two different circumstances (different times and/or places)
- It provides a relative change
- It is always positive (because it is obtained from two positive quantities)
- It is a pure number, independent of the unit of measurement in which the quantities are expressed
- It is often used in time-series analysis of a macroeconomic phenomena

Simple Index Number Notation

Given a generic phenomenon X is :

X_0 intensity of X in situation 0

X_1 intensity of X in situation 1

I is an index number

$$ {}_0I_1 = \frac{x_1}{x_0} $$

${}_0I_1$ measures how much the intensity of phenomenon X has changed at 1 (current situation) compared with 0 (base)

0 = base period

1 = current situation

Simple Index Number Interpretation

$$
{}_0I_1 \begin{cases} > 1 \Rightarrow & X_1 \text{ greater than } X_0 \\ = 1 \Rightarrow & X_1 \text{ equal to } X_0 \\ < 1 \Rightarrow & X_1 \text{ less than } X_0 \end{cases}
$$

The Index Number is often expressed as a percentage, i.e. multiplied by 100.

Relative variation and Simple Index Number

$$
\frac{(x_1 - x_0)}{x_0} = \frac{x_1}{x_0} - \frac{x_0}{x_0} = {}_0I_1 - 1
$$

Example: ${}_0I_t=1,09$ $\Rightarrow$ Relative variation = 1,09-1= 0,09

Fixed-base and Mobile-base Index Number

Let us consider the intensities of the phenomenon X
at n different times t .

FIXED-BASE Index Number series   base=0

${}_0I_0\ {}_0I_1\ {}_0I_2 \dots {}_0I_t \dots {}_0I_n$

The base does not vary as «t» varies

MOBILE-BASE Index Number series   base=t-1

${}_0I_0\ {}_0I_1\ {}_1I_2 \dots {}_{t-1}I_t \dots {}_{n-1}I_n$

Mobile-base index numbers measure changes relative to the previous period (short-term variations)

Fixed-Base Index Number

In general, let be

$$X_0,\ X_1,\ X_2,\ X_3,\ \ldots\ X_k,\ \ldots\ X_n$$

the values of a given aggregate at times

$$0, 1, 2, 3, \ldots.., t, \ldots.., n.$$

Assuming time 0 as a fixed base, the index number referring to the generic period t (where t = 1, 2, 3, ….., n) with respect to period 0 will be

$$_0IF_t = \frac{X_t}{X_0} \cdot 100$$

the index numbers with fixed base are calculated taking as reference a fixed value at time 0 that remains constant as time varies, this value is called base (base period).

Mobile-base Index Number

The Mobile-Base index numbers (or chain-linked) are calculated by taking the immediately previous period as a base; they express the percentage variations from one period to the next (period-on-period variations).

$$ {}_{t-1}IM_t = \frac{X_t}{X_{t-1}} \cdot 100 $$

Price Index Numbers
Purpose

Price Index Numbers are of great importance in the economic field.
They are useful for example for:

- evaluate price changes (measurement of inflation);
- evaluate to what extent the variation in an economic aggregate (for example consumption, production, trade, etc.) is partly attributable to the variation in quantities and partly to the variation in the prices of the goods and services that compose them;
- estimate a cost of living indicator;
- obtain a revaluation coefficient (index number) in order to adjust the nominal value of a monetary sum to the current cost of living.

Example

year	t	p_i	Base-fixed index number, 2005=100	Mobile-base index number, base t-1=100
2005	0	1,65	100,0	-
2006	1	1,68	101,8	101,8
2007	2	1,55	93,9	92,3
2008	3	1,65	100,0	106,5
2009	4	2,10	127,3	127,3

1,55/1,65= 0,939

1,55/1,68=0,923

Changing the fixed-base

Changing from one fixed base to another fixed base: to convert a fixed-base index number series into a series with a new fixed base, it is sufficient to divide each index number by the index number for the period chosen as a new base.

Example – Consider a fixed-base index number series (base – year 1999)relating to the divorces in Italy from 1997 to 2001. We want to change into a series with a new fixed-base- year 2000.

	Num. Indici base = 1999			Num. Indici base = 2000
1997	97,1	97,1/109,4 x 100	1997	88,8
1998	97,6	97,6/109,4 x 100	1998	89,2
1999	100,0	100,0/109,4 x 100	1999	91,4
2000	109,4	109,4/109,4 x 100	2000	100,0
2001	116,6	116,6/109,4 x 100	2001	106,6

The relationship between fixed-base and mobile-base index numbers is such that a fixed-base index number (base 0) for the period t is the product of the mobile-base index number series from 1 to t:

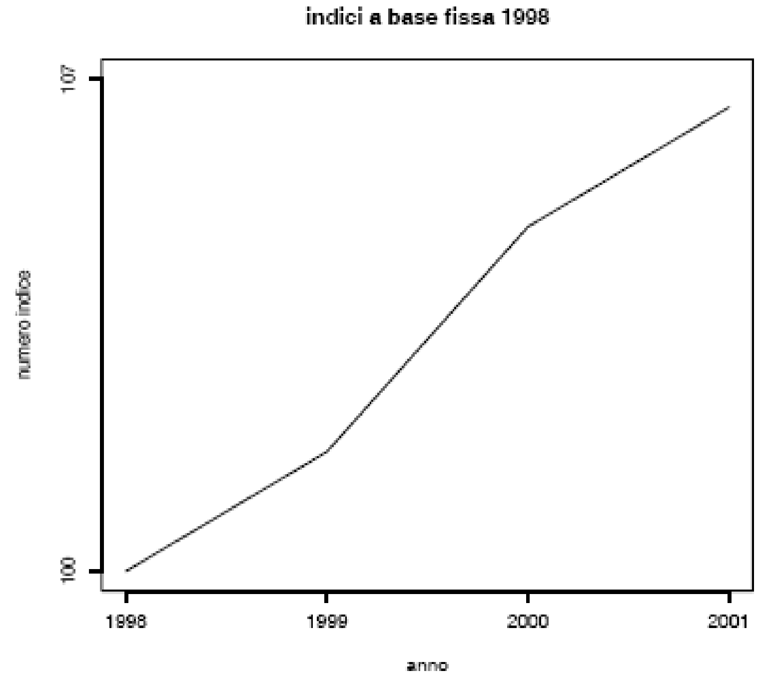
$$(IM_1 \cdot IM_2 \cdots IM_{t-1} \cdot IM_t) \cdot 100 =$$

$$= \left(\frac{X_1}{X_0} \cdot \frac{X_2}{X_1} \cdots \frac{X_{t-1}}{X_{t-2}} \cdot \frac{X_t}{X_{t-1}}\right) \cdot 100 =$$

$$= \frac{X_t}{X_0} \cdot 100 =_0 IF_t$$

Example: The table shows the Gross Domestic Product (GDP) series from 1998 to 2001 at constant 1998 prices.

anno	PIL (in milioni di euro)	Indici a base fissa (1998=100)
1998	968681	100.0
1999	984713	101.7
2000	1015845	104.9
2001	1032818	106.6



→ Identity property

In the example on GDP we change the fixed-base index number using the year 1999.

anno	PIL (in milioni di euro)	Indici a base fissa (1999=100)
1998	968681	98.4
1999	984713	100.0
2000	1015845	103.2
2001	1032818	104.9

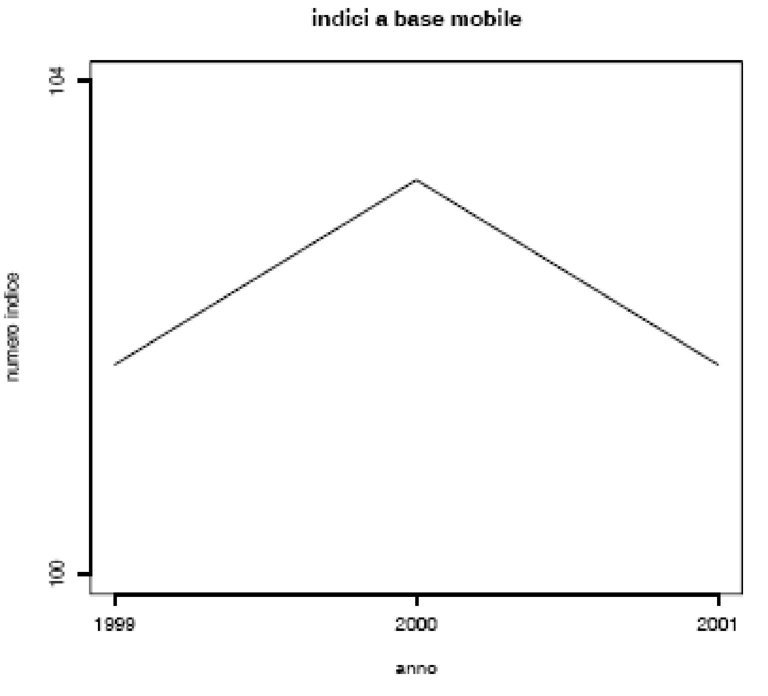
anno	PIL (in milioni di euro)	Indici a base fissa (1998=100)
1998	968681	100.0
1999	984713	101.7
2000	1015845	104.9
2001	1032818	106.6

$$ {}_{1999}I_{1998} = 0.984 = \frac{1}{1.017} = \frac{1}{{}_{1998}I_{1999}}. $$

→ Bases Reversibility property

Considering the example on GDP:

anno	PIL (in milioni di euro)	Indici a base mobile
1998	968681	–
1999	984713	101.7
2000	1015845	103.2
2001	1032818	101.7



→ relationship between mobile-base index numbers and fixed-base index number

Composite Index Number

It is a particular ratio (or index number) that simultaneously and synthetically measures the variations of a set of quantities observed in two different circumstances (different times and/or places). For example the variation in prices referring to several goods/services; the variation in production referring to several goods/services of the different sectors of activity.

The components of the index number are all of the same type and result in a synthetic index number that measures simultaneously the variation in the quantities considered. For example, the consumer price index measures the variation in the general price level for a category of goods intended for consumption.

In general, composite index numbers are used for temporal (or spatial) comparisons of prices, quantities (produced, consumed, imported, etc.) or, finally, monetary values. In general terms, such measures refer to a distinct set of goods/services.”

Composite Index Number
Definition and notation

Suppose we have a set of **prices referring to n goods/services** that we denote by p. Each price is observed over time by 0, 1, 2, ..., T .

If we want to measure the variation over time of different prices as a whole, we need to calculate a complex index number, or simply, a price index number.

If the prices refer to n products purchased by households for consumption, we obtain a consumer price index.

The intensity of the phenomenon p is defined by two labels: *i* identifies the generic consumer good and *t* the period, i.e. $p_{i,t}$.

Suppose we want to define a synthetic index for a set of n goods and services for which the sets of prices *p* and quantities *q* at times 0, 1, 2, … , t, …, T are defined:

Beni	**Prezzi**	**Quantità**
1	$p_{10}, \dots, p_{1t}, \dots, p_{1T}$	$q_{10}, \dots, q_{1t}, \dots, q_{1T}$
2	$p_{20}, \dots, p_{2t}, \dots, p_{2T}$	$q_{20}, \dots, q_{2t}, \dots, q_{2T}$
…		
k	$p_{k0}, \dots, p_{kt}, \dots, p_{kT}$	$q_{k0}, \dots, q_{kt}, \dots, q_{kT}$
…		
n	$p_{n0}, \dots, p_{nt}, \dots, p_{nT}$	$q_{n0}, \dots, q_{nt}, \dots, q_{nT}$

The synthetic price index will be based on the following columns:

$$I_{0t} = \frac{F(p_{1t}, \ \dots \ , p_{nt})}{G(p_{10}, \ \dots \ , p_{n0})}$$

where F and G are two price aggregation functions

goods	**prices**	**quantities**
1	p_{10}, … , p_{1t}, … , p_{1T}	q_{10}, … , q_{1t}, … , q_{1T}
2	p_{20}, … , p_{2t}, … , p_{2T}	q_{20}, … , q_{2t}, … , q_{2T}
…		
k	p_{k0}, … , p_{kt}, … , p_{kT}	q_{k0}, … , q_{kt}, … , q_{kT}
…		
n	p_{n0}, … , p_{nt}, … , p_{nT}	q_{n0}, … , q_{nt}, … , q_{nT}

$$I_{0t} = \frac{F(p_{1t}, \; \dots \; , p_{nt})}{G(p_{10}, \; \dots \; , p_{n0})}$$

F and G are two price aggregation functions;

- It can be sostitued by an economic value such as the product between quantities and prices.

- If we multiply prices by quantities it means we calculate spending values

- the synthetic price index is thus obtained as a ratio of two values of expenditures

Laspeyres Price Index

If the weighting structure is fixed and the quantities consumed are those at initial time 0, we have the **Laspeyres Price Index**.

goods	Prices	quantities
1	$p_{10}, \ldots, p_{1t}, \ldots, p_{1T}$	$q_{10}, \ldots, q_{1t}, \ldots, q_{1T}$
2	$p_{20}, \ldots, p_{2t}, \ldots, p_{2T}$	$q_{20}, \ldots, q_{2t}, \ldots, q_{2T}$
…		
k	$p_{k0}, \ldots, p_{kt}, \ldots, p_{kT}$	$q_{k0}, \ldots, q_{kt}, \ldots, q_{kT}$
…		
n	$p_{n0}, \ldots, p_{nt}, \ldots, p_{nT}$	$q_{n0}, \ldots, q_{nt}, \ldots, q_{nT}$

$$I_{0t} = \frac{F(p_{1t}, \ldots, p_{nt})}{G(p_{10}, \ldots, p_{n0})}$$

$$G(p_{10}, \ldots, p_{n0}) = \sum_{k=1}^{n} p_{k0} q_{k0} \qquad F(p_{1t}, \ldots, p_{nt}) = \sum_{k=1}^{n} p_{kt} q_{k0}$$

goods	**prices**	**quantities**
1	$p_{10}, \ldots, p_{1t}, \ldots, p_{1T}$	$q_{10}, \ldots, q_{1t}, \ldots, q_{1T}$
2	$p_{20}, \ldots, p_{2t}, \ldots, p_{2T}$	$q_{20}, \ldots, q_{2t}, \ldots, q_{2T}$
…		
k	$p_{k0}, \ldots, p_{kt}, \ldots, p_{kT}$	$q_{k0}, \ldots, q_{kt}, \ldots, q_{kT}$
…		
n	$p_{n0}, \ldots, p_{nt}, \ldots, p_{nT}$	$q_{n0}, \ldots, q_{nt}, \ldots, q_{nT}$

$$I_{0t} = \frac{F(p_{1t}, \ldots, p_{nt})}{G(p_{10}, \ldots, p_{n0})}$$

$$G(p_{10}, \ldots, p_{n0}) = \sum_{k=1}^{n} p_{k0} q_{k0} \qquad F(p_{1t}, \ldots, p_{nt}) = \sum_{k=1}^{n} p_{kt} q_{k0}$$

goods	Prices	quantities
1	$p_{10}, \ldots, p_{1t}, \ldots, p_{1T}$	$q_{10}, \ldots, q_{1t}, \ldots, q_{1T}$
2	$p_{20}, \ldots, p_{2t}, \ldots, p_{2T}$	$q_{20}, \ldots, q_{2t}, \ldots, q_{2T}$
…		
k	$p_{k0}, \ldots, p_{kt}, \ldots, p_{kT}$	$q_{k0}, \ldots, q_{kt}, \ldots, q_{kT}$
…		
n	$p_{n0}, \ldots, p_{nt}, \ldots, p_{nT}$	$q_{n0}, \ldots, q_{nt}, \ldots, q_{nT}$

Laspeyres Price Index

$$
{}_{p}I_{0t}^{L} = \frac{F(p_{1t}, \ \ldots \ , p_{nt})}{G(p_{10}, \ \ldots \ , p_{n0})} = \frac{\sum_{k=1}^{n} p_{kt} q_{k0}}{\sum_{k=1}^{n} p_{k0} q_{k0}}
$$

Esempio

- Supponiamo che un'impresa di pulizie voglia costruire un indice dei prezzi complesso relativo al prezzo unitario in lire dei prodotti casalinghi utilizzati negli anni 1994-1996 per svolgere la propria attività

	Prezzo unitario		
	Detergenti per vetri	Cera per pavimenti	Scopa
	bene 1	bene 2	bene 3
1994	2.500	5.300	10.500
1995	2.700	5.800	11.500
1996	3.000	6.000	12.000

Indice dei prezzi di Laspeyres

Se la struttura della ponderazione è fissa e le quantità consumate sono quelle al tempo iniziale 0, abbiamo ***l'indice di Laspeyres***.

	Prezzo unitario				Quantità		
	Detergenti per vetri	Cera per pavimenti	Scopa		Detergenti per vetri	Cera per pavimenti	Scopa
	bene 1	bene 2	bene 3		bene 1	bene 2	bene 3
1994	2.500	5.300	10.500	1994	200	150	35
1995	2.700	5.800	11.500				
1996	3.000	6.000	12.000				

Dal punto di vista formale abbiamo che:

	Prezzo unitario				Quantità		
	Detergenti per vetri	Cera per pavimenti	Scopa		Detergenti per vetri	Cera per pavimenti	Scopa
	bene 1	bene 2	bene 3		bene 1	bene 2	bene 3
1994	2.500	5.300	10.500	1994	200	150	35
1995	2.700	5.800	11.500				
1996	3.000	6.000	12.000				

$$
{}_{1994}I^{L}_{1995} = \frac{\sum_{i=1}^{3} p_{i1995} \cdot q_{i1994}}{\sum_{i=1}^{3} p_{i1994} \cdot q_{i1994}} = \frac{(p_{11995} \cdot q_{11994}) + (p_{21995} \cdot q_{21994}) + (p_{21995} \cdot q_{21994})}{(p_{11994} \cdot q_{11994}) + (p_{21994} \cdot q_{21994}) + (p_{21994} \cdot q_{21994})} =
$$

$$
= \frac{(2.700 \times 200) + (5.800 \times 150) + (11.500 \times 35)}{(2.500 \times 200) + (5.300 \times 150) + (10.500 \times 35)} = \frac{1.812.500}{1.662.500} = 109{,}0
$$

$$
{}_{0}I^{L}_{t} = \frac{\sum_{i=1}^{k} p_{it} \cdot q_{io}}{\sum_{i=1}^{k} p_{i0} \cdot q_{i0}}
$$

Questa vale per un generico anno t su un anno base 0

Paasche Price Index

An alternative to the Laspeyres price index is the Paasche price index in which the quantities of the final moment t are instead kept fixed

Beni	**Prezzi**	**Quantità**
1	$p_{10}, \dots, p_{1t}, \dots, p_{1T}$	$q_{10}, \dots, q_{1t}, \dots, q_{1T}$
2	$p_{20}, \dots, p_{2t}, \dots, p_{2T}$	$q_{20}, \dots, q_{2t}, \dots, q_{2T}$
…		
k	$p_{k0}, \dots, p_{kt}, \dots, p_{kT}$	$q_{k0}, \dots, q_{kt}, \dots, q_{kT}$
…		
N	$p_{n0}, \dots, p_{nt}, \dots, p_{nT}$	$q_{n0}, \dots, q_{nt}, \dots, q_{nT}$

Paasche Price Index

$$ {}_{p}I_{0t}^{P} = \frac{F(p_{1t}, \dots, p_{nt})}{G(p_{10}, \dots, p_{n0})} = \frac{\sum_{k=1}^{n} p_{kt} q_{kt}}{\sum_{k=1}^{n} p_{k0} q_{kt}} $$

Notation of a composite index numbers

if we measure changes in price (p), quantity (q) or values (v)

the formula used to obtain the synthetic price or quantity index (L,P,F)

$$ {}^{p}_{0}I^{L}_{t} $$

base period

Current or reference period

Esempio 1: prezzi medi di 3 beni in 3 tempi

	2009	2010	2011
Bene 1	1,5	1,8	1,9
Bene 2	6,3	6,5	7
Bene 3	3,5	3,8	4,2

Utilizzando numeri indici elementari possiamo misurare la variazione di prezzo dei singoli beni.
Per sintetizzare la variazione dei prezzi dei 3 beni occorre calcolare un *indice sintetico dei prezzi*.

Calcolo di numeri indici semplici

	2009	2010	2011
Bene 1	100%	120%	127%
Bene 2	100%	103%	111%
Bene 3	100%	109%	120%

Es: tra il 2009 e il 2010 il prezzo del bene 1 aumenta del 20%, il prezzo del bene 2 del 3% e il prezzo del bene 3 del 9%.

Di quanto varia il prezzo dell'insieme dei tre beni?

Riprendiamo i dati dell'Esempio 1, affiancando ai prezzi dei tre beni, le quantità acquistate nei tre periodi considerati.

	2005		2006		2007	
	prezzi	quantità	prezzi	quantità	prezzi	quantità
bene1	1,5	11	1,8	7,5	1,9	4
bene2	6,3	10	6,5	8	7	6
bene3	3,5	12	3,8	10	4,2	6,5

Come possiamo definire i pesi w_i, ovvero l'importanza di ciascun bene nel calcolo dell'indice sintetico dei prezzi?

ESERCIZIO - Riprendiamo l'esempio e calcoliamo per il 2007 con base 2005: i) gli indici dei prezzi e delle quantità secondo L, P;
ii) l'indice di valore per la spesa complessiva

TABELLA 3

	2005		2006		2007	
	prezzi	quantità	prezzi	quantità	prezzi	quantità
bene1	1,5	11	1,8	7,5	1,9	4
bene2	6,3	10	6,5	8	7	6
bene3	3,5	12	3,8	10	4,2	6,5

Per fini di calcolo conviene far riferimento alle formule degli indici viste come RAPPORTI DI AGGREGATI e quindi impostare il calcolo come nella tabella che segue:

	$p_{05}q_{05}$	$p_{05}q_{07}$	$p_{07}q_{05}$	$p_{07}q_{07}$
bene1	16.5	6	20.9	7.6
bene2	63	37.8	70	42
bene3	42	22.75	50.4	27.3
Totale	121.5	66.55	141.3	76.9

I diversi numeri indici possono essere calcolati facilmente utilizzando i totali di tabella. Ad esempio:

$$\boxed{{}_{05}^{p}I_{07}^{L} = \frac{\sum p_{07} \cdot q_{05}}{\sum p_{05} \cdot q_{05}} = \frac{141.3}{121.5} = 1.163}$$

$$\boxed{{}_{05}^{p}I_{07}^{P} = \frac{\sum p_{07} \cdot q_{07}}{\sum p_{05} \cdot q_{07}} = 1.156}$$

Soluzioni

$$ {}_{05}^{p}I_{07}^{L} = \frac{\sum p_{07} \cdot q_{05}}{\sum p_{05} \cdot q_{05}} = 1.163 $$

$$ {}_{05}^{q}I_{07}^{L} = \frac{\sum p_{05} \cdot q_{07}}{\sum p_{05} \cdot q_{05}} = 0.548 $$

$$ {}_{05}^{p}I_{07}^{P} = \frac{\sum p_{07} \cdot q_{07}}{\sum p_{05} \cdot q_{07}} = 1.156 $$

$$ {}_{05}^{q}I_{07}^{P} = \frac{\sum p_{07} \cdot q_{07}}{\sum p_{07} \cdot q_{05}} = 0.544 $$

$$ {}_{05}^{p}I_{07}^{F} = \sqrt{1.163 \cdot 1.156} = 1.159 $$

$$ {}_{05}^{q}I_{07}^{F} = \sqrt{0.548 \cdot 0.544} = 0.546 $$

$$ {}_{05}^{v}I_{07} = \frac{76.9}{121.5} = 0.633 $$

Tra il 2005 e il 2007, il valore dell'aggregato ha registrato una diminuzione pari a circa il 37%. Essa è il risultato di una riduzione delle quantità (o volume) dei prodotti acquistati (circa -45.5%) e di un incremento di prezzo del paniere pari a circa il 16%.

Main Index numbers produced by ISTAT

Industrial activity
Services
Prices
Trade interchange with abroad
Labour market
National accounting

The criteria followed by Istat for the collection of basic data and for the construction of the indices can be found on the website www.istat.it attached to the *periodic press releases*.

Main composite index numbers calculated by ISTAT (Value, Price and Quantity Index)

ISTAT is responsible for periodically processing index numbers documenting the trends over time of various aspects of economic life in Italy. These can be grouped into four classes: prices, production, trade and labour.

Prices

- Wholesale and producer price index numbers
- Consumer price index numbers

Industrial activity

-Index numbers of agricultural and forestry production

-Index numbers of industrial production

-Index numbers of turnover, orders and firmness of orders of industry

Trade

- Index numbers of sales of fixed retail trade of the medium and large distributive trades
- Index numbers of quantities, prices and value of imports and exports

Labour market

- Index numbers of contractual and hourly earnings per employee, gross earnings and labour costs.

The criteria followed by Istat for the collection of basic data and the construction of the indices are illustrated in detail in the series «Methods and Standards» as well as in the «Information Note» circulated at the time of publication of the indices and indicates in detail the updates of the basket, the weighting structure and the survey.

For Italy, ISTAT publishes several price indices:

- *The price index of products purchased by farmers*
- *The producer price indices*
- *The cost indices of construction of construction products*
- *The consumer price indices*